

# Unification of inflation, dark energy, and dark matter within the Salam-Sezgin cosmological model

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We investigate a cosmological model, based on the Salam-Sezgin six-dimensional supergravity theory and on previous work by Anchordoqui, Goldberg, Nawata, and Nuñez. Assuming a period of warm inflation, we show that it is possible to extend the evolution of the model back in time, to include the inflationary period, thus unifying inflation, dark matter, and dark energy within a single framework. Like the previous authors, we were not able to obtain the full dark matter content of the Universe from the Salam-Sezgin scalar fields. However, even if only partially successful, this work shows that present-day theories, based on superstrings and supergravity, may eventually lead to a comprehensive modelling of the evolution of the Universe. We find that the gravitational-wave spectrum of the model has a non-constant negative slope in the frequency range  $(10^{-15} - 10^6)$  rad/s, and that, unlike standard (cold) inflation models, it shows no structure in the MHz/GHz range of frequencies.

## I. INTRODUCTION

We still are a long way from having a theoretically sound unified model explaining both the inflationary epoch and the standard era of expansion. The problem has become even more complex since the discovery of the dark energy content and the resulting present accelerated expansion of the Universe [1, 2]. Recently, Anchordoqui, Goldberg, Nawata, and Nuñez [3] proposed a very interesting model based on the Salam-Sezgin six-dimensional supergravity model [4], whose spontaneous compactification from six to four dimensions gives rise to a potential, henceforth called the Salam-Sezgin potential. When this model is lifted to M-theory, the internal space is found to be non-compact [5], circumventing existing no-go theorems and allowing the potential to be positive. The work of Anchordoqui *et al.* [3] is limited to the investigation of the epochs subsequent to primordial nucleosynthesis. The authors derived a solution of the Einstein field equations in qualitative agreement with the observations of an accelerating universe, where the dark energy comes from a scalar field slowly rolling down the (exponential) Salam-Sezgin potential. Another important result was the finding that the model also contains a cold dark matter component ( $p = 0$ ), coming from another scalar field interacting with the dark energy field, whose particles, as

a result of this interaction, have an effective mass slowly evolving with the dark-energy scalar field. Unfortunately, dark matter from the Salam-Sezgin scalar field only accounts for up to about 6% of the total matter content of the universe.

In the present paper we extend this model back in time. It has been known for a long time [6] that an exponential potential of the generic form  $V = V_0 \exp(-\sqrt{8\pi G}\gamma\phi)$  is able to support inflation only if  $\gamma < \sqrt{2}$ . The Salam-Sezgin potential fails to obey this requirement, having  $\gamma = \sqrt{2}$ . But, if we assume a period of warm inflation [7], it is possible to show that the Salam-Sezgin potential is able to drive enough inflation. We obtain, therefore, within the Salam-Sezgin cosmological model, a triple unification of inflation, dark matter, and dark energy. As in Ref. [3], we were not able to obtain the full dark matter content of the Universe from the Salam-Sezgin scalar fields, the rest of the dark matter and the baryonic matter having to be put in by hand. Even if only partially successful, we believe that our analysis shows that present day theories, based on supergravity and superstrings, may eventually be successful in providing a comprehensive description of the evolution of the Universe.

During the period of warm inflation, energy is continuously transferred from the Salam-Sezgin scalar fields to a radiation fluid due to the presence of dissipative terms in the energy conservation equations [7] (for a review see [8] and references therein). For illustrative purposes we will assume a simple phenomenological dissipative term, following the suggestion of Yokoyama and Maeda [9]. In this work we will not attempt to justify the occurrence

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of warm inflation, which should be due to the coupling of the scalar fields to a heat bath of background fields [10]. We just mention that plenty of such fields are present if the model is assumed to be derived from string theory.

In the next section we briefly review the Salam-Sezgin model. In Sect. III we derive the equations defining the model and introduce the details of how, following the inflationary period, we calculate the amount of dark energy and dark matter. This is followed by a section on the numerical simulations of the evolution of the universe, since the beginning of the inflationary period till the present time, and the analysis of the results obtained. Section V is dedicated to the calculation of the full energy spectrum of the gravitational waves generated during the expansion of the universe, as it would be measured today by an ideal detector. Finally, we end with the summary and the conclusions.

## II. THE SALAM-SEZGIN POTENTIAL

We begin with a review of the Salam-Sezgin model, following the notation of Anchordoqui *et al.* [3]. The Salam-Sezgin model represents a six-dimensional supergravity [4]. It has been shown by Cvetič, Gibbons, and Pope [5] that this model can be uplifted consistently to type I superstring theory. Its bosonic part is given by the action

$$S = \frac{1}{4\kappa^2} \int d^6x \sqrt{g_6} \left[ R - \kappa^2 (\partial_M \sigma)^2 - \frac{2g^2}{\kappa^2} e^{-\kappa\sigma} - \kappa^2 e^{\kappa\sigma} F_{MN}^2 - \frac{\kappa^2}{3} e^{2\kappa\sigma} G_{MNP}^2 \right]. \quad (1)$$

Here,  $g_6 = \det g_{MN}$ ,  $R$  is the Ricci scalar of  $g_{MN}$ ,  $\sigma$  is a scalar field,  $F_{MN} = \partial_{[M} A_{N]}$ ,  $G_{MNP} = \partial_{[M} B_{NP]} + \kappa A_{[M} F_{NP]}$ , where  $A_N$  is a gauge field and  $B_{NP}$  is the Kalb-Ramond field; capital Latin indices run from 0 to 5. Introducing  $G_6 \equiv 2\kappa^2$  and  $\xi \equiv 4g^2$  and rescaling  $\phi \equiv -\kappa\sigma$  leads to

$$S = \frac{1}{2G_6} \int d^6x \sqrt{g_6} \left[ R - (\partial_M \phi)^2 - \frac{\xi}{G_6} e^\phi - \frac{G_6}{2} e^{-\phi} F_{MN}^2 - \frac{G_6}{6} e^{-2\phi} G_{MNP}^2 \right]. \quad (2)$$

Here the length dimensions are:  $[G_6] = L^4$ ,  $[\xi] = L^2$ ,  $[\phi] = [g_{MN}^2] = 1$ ,  $[A_M^2] = L^{-4}$ , and  $[F_{MN}^2] = [G_{MNP}^2] = L^{-6}$ .

Salam and Sezgin [4] showed that Lagrangian (1) can be compactified on the direct product  $M = M_4 \times S_2$ , where  $M_4 \equiv (\text{Minkowski})_4$ . It was shown by Gibbons, Güven, and Pope [11] that this is the unique ground state among all non-singular solutions with a four-dimensional Poincaré, de Sitter or anti-de Sitter symmetry. The monopole configuration they found has the metric on  $M$  locally of the form

$$ds_6^2 = ds_4(t, \vec{x})^2 + e^{2f(t, \vec{x})} r_c^2 (d\vartheta^2 + \sin^2 \vartheta d\varphi^2), \quad (3)$$

where  $(t, \vec{x})$  denotes a local coordinate system on  $M_4$  and  $r_c$  is the compactification radius of  $S_2$ . The scalar field  $\phi$  is taken to depend only on the point of  $M_4$ , i.e.,  $\phi = \phi(t, \vec{x})$ . The gauge field  $A_M$  is excited on  $S_2$  and is of the form

$$A_\vartheta = 0, \quad A_\varphi = b \cos \vartheta, \quad (4)$$

which satisfies the Maxwell equations (obtained by varying  $A_M$ ) and yields the field strength

$$F_{MN}^2 = 2b^2 e^{-4f} / r_c^4. \quad (5)$$

Finally  $B_{NP}$  is taken to vanish. It follows that also  $G_{MNP} = 0$ .

The Ricci scalar can now be written as [12]

$$R[M] = R[M_4] + e^{-2f} R[S_2] - 4\Box f - 6(\partial_\mu f)^2, \quad (6)$$

where  $R[M]$ ,  $R[M_4]$ , and  $R[S_2]$  denote the Ricci scalars of the manifolds  $M$ ,  $M_4$ , and  $S_2$ , respectively. It follows

$$R[S_2] = 2/r_c^2 \quad (7)$$

and  $\sqrt{g_6} = e^{2f} r_c^4 \sqrt{g_4}$ , where  $g_4 = \det g_{\mu\nu}$  (with Greek indices running from 0 to 3). We now define the gravitational constant in four dimensions as

$$G_4 \equiv \frac{1}{m_p^2} = \frac{G_6}{32\pi^2 r_c^2}. \quad (8)$$

It follows that by using the Salam-Sezgin monopole configuration we can re-write the action in Eq. (2) as follows

$$S = \frac{1}{16\pi G_4} \int d^4x \sqrt{g_4} \left\{ e^{2f} \left[ R[M_4] + \frac{2}{r_c^2} e^{-2f} + 2(\partial_\mu f)^2 - (\partial_\mu \phi)^2 \right] - \frac{\xi}{G_6} e^{2f+\phi} - \frac{G_6 b^2}{r_c^4} e^{-2f-\phi} \right\}. \quad (9)$$

Using the rescaling of the metric  $\hat{g}_{\mu\nu} \equiv e^{2f} g_{\mu\nu}$  and  $\sqrt{\hat{g}_4} = e^{4f} \sqrt{g_4}$ , the model is taken into the Einstein conformal frame where the action Eq. (9) takes the form

$$S = \frac{1}{16\pi G_4} \int d^4x \sqrt{\hat{g}_4} \left[ R[\hat{g}_4] - 4(\partial_\mu f)^2 - (\partial_\mu \phi)^2 - \frac{\xi}{G_6} e^{-2f+\phi} - \frac{G_6 b^2}{r_c^4} e^{-6f-\phi} + \frac{2}{r_c^2} e^{-4f} \right]. \quad (10)$$

The four-dimensional Lagrangian is then

$$L = \frac{\sqrt{g}}{16\pi G} [R - 4(\partial_\mu f)^2 - (\partial_\mu \phi)^2 - V(f, \phi)], \quad (11)$$

with

$$V(f, \phi) \equiv \frac{\xi}{G_6} e^{-2f+\phi} + \frac{G_6 b^2}{r_c^4} e^{-6f-\phi} - \frac{2}{r_c^2} e^{-4f}, \quad (12)$$

where we have written  $g \equiv \hat{g}_4$ ,  $R \equiv R[\hat{g}_4]$  and  $G \equiv G_4$ .

Following Anchordoqui *et al.* [3], we now perform the field redefinition

$$x \equiv (\phi + 2f)/\sqrt{16\pi G}, \quad y \equiv (\phi - 2f)/\sqrt{16\pi G}. \quad (13)$$

The Lagrangian becomes

$$L = \sqrt{g} \left[ \frac{R}{16\pi G} - \frac{1}{2}(\partial x)^2 - \frac{1}{2}(\partial y)^2 - V(x, y) \right], \quad (14)$$

with the Salam-Sezgin potential given by

$$V(x, y) = A_1 e^{\alpha y} (1 - 2A_2 e^{-\alpha x} + A_3 e^{-2\alpha x}), \quad (15)$$

where we have introduced the notation

$$A_1 = \frac{\xi m_p^4}{512\pi^3 r_c^2}, \quad A_2 = \frac{32\pi^2}{\xi m_p^2}, \quad A_3 = \frac{1024\pi^4 b^2}{\xi m_p^4}, \quad (16)$$

and  $\alpha = \sqrt{16\pi G} = 4\sqrt{\pi}/m_p$ .

### III. THE EVOLUTION MODEL

In our analysis of the Salam-Sezgin cosmological model, we will divide the evolution of the universe in two stages. The first stage starts at the beginning of the inflationary period and extends well into the radiation-dominated epoch. During this first stage of evolution, energy is continuously transferred from the scalar fields  $x$  and  $y$  to a radiation fluid due to the presence of dissipative terms in the energy conservation equations. Inflation, which we assume to be of the warm type [7], is driven by the field  $x$ . When the energy density of radiation is of the order  $(10^{14} \text{ GeV})^4$  (see Sect. IV for details), we assume that the energy transfer from the fields  $x$  and  $y$  to radiation ceases. This marks the end of the first stage of evolution. During the second stage, which extends up to the present epoch, the  $x$ -field oscillates around the minimum of the potential, behaving as cold dark matter with varying mass, as explained below. The  $y$ -field, which behaves like dark energy, is practically constant during most of the second stage of evolution and becomes dominant in a recent epoch, in agreement with observations. In what follows we present the relevant equations for both stages of evolution and show how an unification of inflation, dark matter, and dark energy can be achieved within the Salam-Sezgin cosmological model.

For a flat universe, described by the Friedmann-Robertson-Walker metric,

$$ds^2 = -dt^2 + a^2(t)d\chi^2, \quad (17)$$

where  $a(t)$  is the scale factor and  $d\chi^2$  is the metric of the three-dimensional Euclidean space, the Einstein equations are given by

$$\frac{\ddot{a}}{a} = -\frac{\alpha^2}{6} [\dot{x}^2 + \dot{y}^2 - V(x, y) + \rho_r], \quad (18)$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\alpha^2}{6} \left[ \frac{\dot{x}^2}{2} + \frac{\dot{y}^2}{2} + V(x, y) + \rho_r \right], \quad (19)$$

where  $V(x, y)$  is the Salam-Sezgin potential given by Eq. (15),  $\rho_r$  is the energy density of a radiation fluid with equation of state  $p_r = \rho_r/3$ , and a dot denotes a derivative with respect to the comoving time  $t$ .

Let us assume that, during the first stage of evolution, energy is continuously transferred from the scalar fields  $x$  and  $y$  to the radiation fluid  $\rho_r$  due to the presence of dissipative terms. In this case, the energy conservation equations for the scalar fields and the radiation fluid are

$$\ddot{x} + 3\frac{\dot{a}}{a}\dot{x} + \frac{\partial V}{\partial x} = -\Gamma_x \dot{x}, \quad (20)$$

$$\ddot{y} + 3\frac{\dot{a}}{a}\dot{y} + \frac{\partial V}{\partial y} = -\Gamma_y \dot{y}, \quad (21)$$

$$\dot{\rho}_r + 4\frac{\dot{a}}{a}\rho_r = \Gamma_x \dot{x}^2 + \Gamma_y \dot{y}^2, \quad (22)$$

where  $\Gamma_x$  and  $\Gamma_y$  are the dissipative coefficients.

The form of the dissipative coefficients has been discussed in literature and can be, in realistic models, quite complicated. For illustrative purposes we will consider in this article simple phenomenological dissipative coefficients, first introduced by Yokoyama and Maeda [9], of the form

$$\Gamma_x(x, y) = f_x \sqrt{\frac{\partial^2 V}{\partial x^2}}, \quad (23)$$

$$\Gamma_y(x, y) = f_y \sqrt{\frac{\partial^2 V}{\partial y^2}}, \quad (24)$$

where  $f_x$  and  $f_y$  are free parameters. For an appropriate choice of these parameters, a significant amount of energy is transferred from the scalar fields  $x$  and  $y$  to the radiation fluid and, as a consequence, the energy density of radiation is not diluted during inflation, as in the standard (cold) inflationary models, and its influence in the evolution of the universe is such that enough inflation occurs, despite the steepness of the Salam-Sezgin potential.

To be general, let us assume that, immediately prior to the inflationary phase, the universe may have, in addition to the energy contained in the  $x$  and  $y$  scalar fields, an important contribution from the radiation fluid. Eventually, the potential energy  $V(x, y)$  becomes dominant and warm inflation begins. Initially, the energy density of radiation,  $\rho_r$ , decreases faster than the energy density of the fields  $x$  and  $y$ ,  $\rho_{xy} = \dot{x}^2/2 + \dot{y}^2/2 + V(x, y)$ . However, because of the energy transfer due to the dissipative terms,  $\rho_r$  quickly reaches a state during which it decreases slower than  $\rho_{xy}$ . At a certain point,  $\rho_r$  becomes greater than  $\rho_{xy}$  and a smooth transition from warm inflation to a radiation-dominated universe takes place [13].

During inflation the  $x$ -field slowly rolls down the potential  $V(x, y)$ , approaching its minimum at  $x_{\min} = (1/\alpha) \ln(A_3/A_2)$ . During the inflationary period the  $y$ -field does not play a significant role in the evolution of the universe; it simply slowly rolls down the exponential (in the  $y$  direction) potential.

Note that the potential (15) can be expanded near its minimum, yielding

$$V(x, y) = Ae^{\alpha y} + \frac{1}{2}M_x^2(x - x_{\min})^2 + \dots, \quad (25)$$

where

$$M_x = \sqrt{2}\alpha \frac{A_1^{1/2}A_2}{A_3^{1/2}}e^{\alpha y/2} \quad (26)$$

is the time-dependent mass of the scalar field  $x$  and

$$A \equiv A_1 \left(1 - \frac{A_2^2}{A_3}\right) = \frac{m_p^4}{512\pi^3 r_c b^2} (b^2 \xi - 1). \quad (27)$$

The first term on the right-hand-side of Eq. (25) behaves like dark energy and, at the present epoch, dominates the dynamics of evolution of the universe. In order to achieve agreement with observations, the constant  $A$  should be very small, of the order of  $10^{-122}m_p^4$  (see Sect. IV for details). This extremely small, but different from zero, value means that supersymmetry should to be broken within the Salam-Sezgin cosmological model [3].

After the end of the inflationary period, energy transfer from the  $x$  and  $y$  fields to the radiation fluid continues. As the universe expands, its temperature drops and eventually the interaction between the scalar fields and the thermal bath becomes negligible. At this point, we turn the dissipative coefficients  $\Gamma_x$  and  $\Gamma_y$  to zero. This marks the end of the first stage of evolution. It is necessary to emphasize here, that the residual amount of energy in the  $x$ -field at the end of the first stage of evolution should be low, in order to make sure that the universe undergoes a long enough radiation-dominated period, ending well after nucleosynthesis [14, 15]. Our numerical simulations (see next section) show that the transition between the first and second stage of evolution occurs at temperatures of the order of  $10^{14}$  GeV and that the scalar field  $x$  is, at that time, of the order of  $-10^{-17}m_p$ .

Due to the absence of the dissipative terms, the scalar field  $x$  oscillates rapidly around its minimum during the second stage of evolution. Since in the expanded potential (25) the dominant term is quadratic, the oscillating  $x$ -field behaves like matter with equation of state  $p = 0$  [16]. This pressureless matter behaves like cold dark matter. Its energy density is proportional to the  $y$ -dependent mass of the  $x$ -field (which is practically constant during most of the second stage of evolution) and decreases as  $a^{-3}$ ,

$$c^2 \rho_x = Ce^{\alpha y/2} \left(\frac{a_0}{a}\right)^3, \quad (28)$$

where  $c$  is the speed of light,  $\alpha = 4\sqrt{\pi G}/c$ , and  $a_0 \equiv a(t_0)$  is the value of the scale factor today<sup>1</sup>. The constant  $C$

should be chosen in order to guarantee continuity of the solution at the transition from the first to the second stage of evolution, namely,

$$C = c^2 \rho_{x,T} e^{-\alpha y_T/2} \left(\frac{\rho_{r,0}}{\rho_{r,T}}\right)^{3/4}, \quad (29)$$

where the subscript  $T$  refers to quantities evaluated at the end of the first stage of evolution and we have taken into account that

$$\rho_r = \rho_{r,0} \left(\frac{a_0}{a}\right)^4, \quad (30)$$

where  $\rho_{r,0} \equiv \rho_r(t_0) = 4.6 \times 10^{-31}$  kg/m<sup>3</sup> is the density of radiation observed today.

Within the Salam-Sezgin cosmological model, the  $x$ -field accounts for just a fraction of the total matter content of the universe [3]. Therefore, in order to be in agreement with observational data, we introduce an additional pressureless matter component, which accounts for other types of dark matter and for the usual (baryonic) matter. For this component, the energy conservation equation yields

$$\rho_m = \frac{B}{c^2} \left(\frac{a_0}{a}\right)^3, \quad (31)$$

where  $B$  is a constant.

The evolution equations for the  $y$ -field and the scale factor  $a$  are now

$$\ddot{y} = -3\frac{\dot{a}}{a}\dot{y} - \alpha Ae^{\alpha y} - \frac{\alpha}{2}Ce^{\alpha y/2} \left(\frac{a_0}{a}\right)^3, \quad (32)$$

$$\frac{\ddot{a}}{a} = -\frac{\alpha^2}{12} \left[ 2\dot{y}^2 - 2Ae^{\alpha y} + \left(B + Ce^{\alpha y/2}\right) \left(\frac{a_0}{a}\right)^3 + 2\rho_{r,0}c^2 \left(\frac{a_0}{a}\right)^4 \right], \quad (33)$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\alpha^2}{6} \left[ \frac{\dot{y}^2}{2} + Ae^{\alpha y} + \left(B + Ce^{\alpha y/2}\right) \left(\frac{a_0}{a}\right)^3 + \rho_{r,0}c^2 \left(\frac{a_0}{a}\right)^4 \right]. \quad (34)$$

In the above equations, the constants  $A$  and  $B$  should be chosen consistently with observations, namely,

$$Ae^{\alpha y_0} + \frac{\dot{y}_0^2}{2} = \rho_{DE,0}c^2, \quad (35)$$

$$B + Ce^{\alpha y_0/2} = \rho_{M,0}c^2, \quad (36)$$

where  $\rho_{DE,0} = 6.9 \times 10^{-27}$  kg/m<sup>3</sup> and  $\rho_{M,0} = 2.6 \times 10^{-27}$  kg/m<sup>3</sup>. To the values of  $\rho_{DE,0}$ ,  $\rho_{M,0}$ , and  $\rho_{r,0}$  used in this article, correspond a value of the Hubble constant  $H_0 = 71$  km s<sup>-1</sup>Mpc<sup>-1</sup>.

Following Ref. [3], during the second stage of evolution, we use, instead of the comoving time  $t$ , a new variable  $u$ , defined as

$$u = -\ln(1+z), \quad (37)$$

<sup>1</sup> In the first stage of evolution we use the natural system of units, with  $m_p = 1/\sqrt{G} = 1.22 \times 10^{19}$  GeV, while in the second stage we use the International Systems of Units.

where  $z = a_0/a - 1$  is the redshift.

Using this new variable, the energy conservation equation for the scalar field  $y$  can be rewritten as

$$y_{uu} = - \left\{ \left[ \frac{\ddot{a}}{a} + 2 \left( \frac{\dot{a}}{a} \right)^2 \right] y_u + \alpha A e^{\alpha y} + \frac{\alpha}{2} C e^{\alpha y/2} e^{-3u} \right\} \left( \frac{\dot{a}}{a} \right)^{-2}, \quad (38)$$

where the subscript  $u$  denotes a derivative with respect to  $u$ ;  $\ddot{a}/a$  and  $(\dot{a}/a)^2$  are functions of  $u$ ,  $y$  and  $y_u$  given by

$$\frac{\ddot{a}}{a} = \frac{\alpha^2}{12} \left\{ 4\alpha^2 \left[ A e^{\alpha y} + \left( B + C e^{\alpha y/2} \right) e^{-3u} + \rho_{r,0} c^2 e^{-4u} \right] y_u^2 (\alpha^2 y_u^2 - 12)^{-1} + 2A e^{\alpha y} - \left( B + C e^{\alpha y/2} \right) e^{-3u} - 2\rho_{r,0} c^2 e^{-4u} \right\}, \quad (39)$$

$$\left( \frac{\dot{a}}{a} \right)^2 = 2\alpha^2 \left[ A e^{\alpha y} + \left( B + C e^{\alpha y/2} \right) e^{-3u} + \rho_{r,0} c^2 e^{-4u} \right] (12 - \alpha^2 y_u^2)^{-1}. \quad (40)$$

The set of equations Eqs. (18)–(22), for the first stage of evolution, and (38)–(40), for the second stage, will be solved numerically in the next section. The parameters  $f_x$  and  $f_y$  will be chosen such that enough inflation (65 or more  $e$ -folds) is achieved during the first stage of evolution. The time of transition between the two stages of evolution will be chosen such that the radiation-dominated epoch taking place after inflation ends well after nucleosynthesis. The constant  $C$  will be chosen such that the solution is continuous at the transition between the first and second stages of evolution and the constants  $A$  and  $B$  will be chosen in order to guarantee that the energy densities of radiation, matter (baryonic plus dark), and dark energy at the present time are in agreement with observations.

In section IV our numerical results will be presented in terms of the density parameters for radiation, matter, and dark energy, the equation-of-state parameter for dark energy, and the deceleration parameter defined as, respectively,

$$\Omega_r = \frac{\rho_r}{\rho_c} = \frac{\alpha^2}{6} \rho_{r,0} c^2 e^{-4u} \left( \frac{\dot{a}}{a} \right)^{-2}, \quad (41)$$

$$\Omega_M = \frac{\rho_M}{\rho_c} = \frac{\alpha^2}{6} \left( B + C e^{\alpha y/2} \right) e^{-3u} \left( \frac{\dot{a}}{a} \right)^{-2}, \quad (42)$$

$$\Omega_y = \frac{\rho_y}{\rho_c} = \frac{\alpha^2}{6} \left[ \frac{y_u^2}{2} + A e^{\alpha y} \left( \frac{\dot{a}}{a} \right)^{-2} \right], \quad (43)$$

where  $\rho_c = 3H^2/(8\pi G) = 6(\dot{a}/a)^2/(\alpha^2 c^2)$  is the critical density,

$$w_y = \frac{p_y}{\rho_y c^2} = \frac{y_u^2 \left( \frac{\dot{a}}{a} \right)^2 - 2A e^{\alpha y}}{y_u^2 \left( \frac{\dot{a}}{a} \right)^2 + 2A e^{\alpha y}}, \quad (44)$$

and

$$q = -\frac{a\ddot{a}}{\dot{a}^2}. \quad (45)$$

To finish this section, let us point out that a triple unification of inflation, dark matter, and dark energy was proposed recently by Liddle, Pahud and Ureña-López [15], using a single massive scalar field  $\phi$  with potential  $V(\phi) = V_0 + \frac{1}{2}m_\phi^2\phi^2$ . Note that this potential is similar to the expanded Salam-Sezgin potential given by Eq. (25), the main difference being the fact that the term behaving like dark energy is constant and not time-dependent as in our case. Within the unification scenario proposed in Ref. [15], a residual inflaton field  $\phi$  survives preheating and oscillates rapidly around the minimum of the potential, behaving as cold dark matter. However, similarly to our case, the amplitude of the scalar field oscillations are too high. They should be drastically reduced in order to allow for a long enough radiation-dominated epoch encompassing primordial nucleosynthesis. Exploiting the uncertainty in the cosmological evolution between the end of the inflationary period and nucleosynthesis, the authors of Ref. [15] considered a brief period of thermal inflation following preheating, in order to reduce the amplitude of the scalar field oscillations to the desired level. In this work, we managed to reduce of the amplitude of the  $x$ -field oscillations after inflation within the context of the Salam-Sezgin cosmological model. As already explained above, this was accomplished by extending the energy transfer from the Salam-Sezgin scalar fields to the radiation fluid until well into the radiation-dominated epoch.

#### IV. NUMERICAL SIMULATIONS

Let us now solve numerically the set of equations (18)–(22) and (38)–(40). As initial conditions, at the beginning of the inflationary period, we choose  $x(t_i) = -0.1 m_P$  and  $y(t_i) = 0.1 m_P$ . Note that enough inflation (65  $e$ -folds or more) can be achieved for such initial amplitudes of the inflaton field  $x$ , below the Planck mass, provided that  $f_x$  is chosen adequately. In what follows we choose the phenomenological dissipative parameters to be  $f_x = f_y = 275$ . In order to achieve agreement with observations, the constant  $A$ , introduced in Eq. (27), should be of the order of  $10^{-122} m_P^4$ . We choose, therefore,  $A_1 = 10^{-12} m_P^4$ ,  $A_2 = 1 - 10^{-110}$ , and  $A_3 = 1$ . This choice of  $A_1$  also fixes the energy scale of inflation to be  $E_{\text{inf}} = V(x_i, y_i)^{1/4} \approx 10^{16} \text{ GeV}$ . We assume that, at the beginning of the inflationary period, the kinetic terms of the scalar fields, as well as the energy density of radiation, are of the same order of magnitude of the potential  $V$ . We choose, therefore,  $\dot{x}(t_i) = \dot{y}(t_i) = 1.47 \times 10^{-6} m_P^2$  and  $\rho_r(t_i) = 4.33 \times 10^{-12} m_P^4$ . Finally, we choose  $a(t_i) = 1$ ;  $\dot{a}(t_i)$  is fixed by the Eq. (19) to be  $8.52 \times 10^{-6} m_P$ .

For such values of the initial conditions and parameters, the inflationary period lasts for about  $1.1 \times 10^8 t_P$

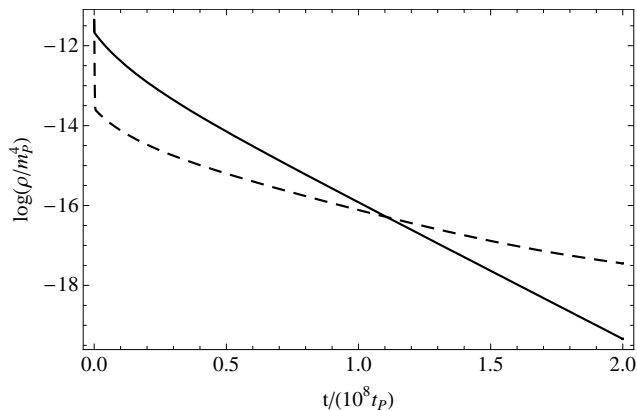


FIG. 1: Time evolution of the energy densities of radiation  $\rho_r$  (dashed curve) and of the scalar fields  $\rho_{xy} = \dot{x}^2/2 + \dot{y}^2/2 + V(x, y)$  during the first stage of evolution. The smooth transition from inflation to a radiation-dominated universe occurs at about  $1.1 \times 10^8 t_P$ .

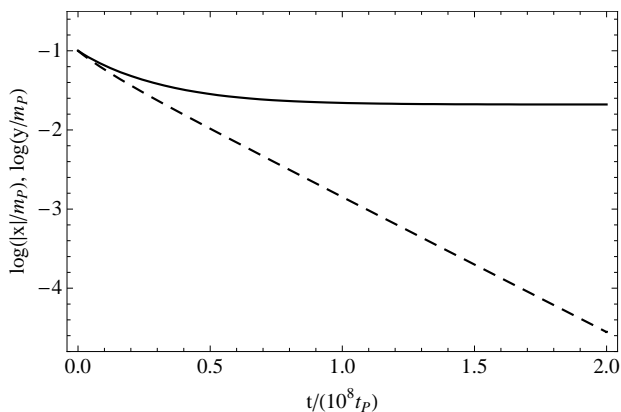


FIG. 2: Time evolution of the scalar fields  $x$  (dashed curve) and  $y$  during the first stage of evolution. After the inflationary period  $y$  decreases very slowly, while the  $|x|$  decreases exponentially.

and the scale factor grows by a factor of about  $10^{28}$ . During inflation the condition  $\rho_r^{1/4} > H$  holds, as expected in warm inflation. Furthermore, the dissipative coefficient  $\Gamma_x$  is much greater than the Hubble parameter  $H$ , implying a strong dissipative warm-inflation regime [8]. At the very beginning of the inflationary period, the energy density of radiation decreases faster than the energy density of the scalar fields. However, because of the energy transfer due to the dissipative terms, the decrease of the energy density of radiation slows down and, after a while, a smooth transition from warm inflation to a radiation-dominated universe takes place (see Fig. 1).

During inflation the  $x$  and  $y$ -fields roll down the Salam-Sezgin potential. At the end of the inflationary period their values are  $x \approx -9.30 \times 10^{-4} m_P$  and  $y \approx 2.16 \times 10^{-2} m_P$ . During the subsequent evolution, till the end of the first stage of evolution,  $y$  decreases very slowly, while  $|x|$  decreases exponentially (see Fig. 2).

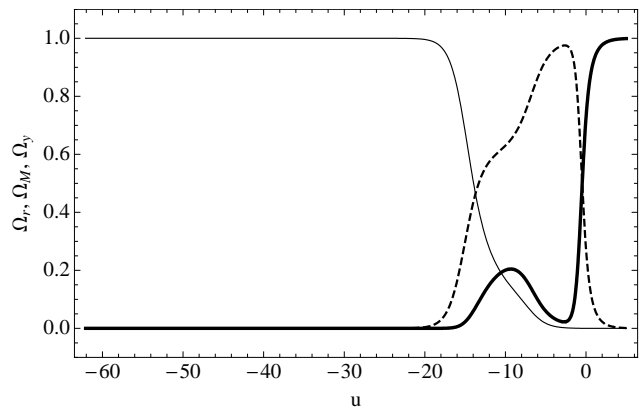


FIG. 3: Time evolution of the density parameters  $\Omega_r$  (thin curve),  $\Omega_M$  (dashed curve), and  $\Omega_y$  (thick curve) during the second stage of evolution. In this case, which corresponds to a short duration of the first stage of evolution,  $8.00 \times 10^8 t_P$ , the transition from a radiation to a matter-dominated universe occurs earlier in the history of the universe (but well after nucleosynthesis) and the density parameter for dark energy becomes non-negligible already at a redshift  $z \approx 10^6$ . The  $x$ -field dark matter amounts to about 10% of the total matter content of the universe at the present time  $u_0 = 0$ .

As already mentioned above, the residual  $x$ -field oscillates rapidly around the minimum of the potential during the second stage of evolution in which  $f_x = f_y = 0$ , behaving like cold dark matter. In order to achieve a long radiation-dominated period of evolution, the initial amplitude of the  $x$ -field oscillations should be much smaller than the Planck mass [14, 15]. Our numerical simulations show that a long enough radiation-dominated epoch requires, at the time  $t_T$  of transition between the first and second stages of evolution, that  $x_T \approx -10^{-17} m_P$ . Therefore, the energy transfer from the scalar fields  $x$  and  $y$  to the radiation fluid should continue after the end of the inflationary period, allowing  $|x|$  to decrease from about  $10^{-3} m_P$ , at the end of inflation, to about  $10^{-17} m_P$ , at the time of transition between the first and second stages of evolution.

The value of  $t_T$  should be chosen very carefully. If this value is too small, the amplitude of the scalar field  $x$  does not decrease enough before it starts to oscillate and, consequently, the transition from a radiation to a matter-dominated universe takes place too soon. Another consequence of a shorter first stage of evolution is a rise of the energy density of the  $y$ -field (dark energy) at redshift  $z \gg 1$ . In Fig. 3, the evolution of radiation, matter (baryonic plus dark), and dark energy during the second stage of evolution is shown for  $t_T = 8.00 \times 10^8 t_P$ . In this case,  $x_T = -1.58 \times 10^{-15} m_P$  and  $\rho_{x,0} = 9.7 \times 10^{-2} \rho_{M,0}$ . On the other hand, if the value of  $t_T$  is too big, the amplitude of the  $x$ -field decreases too much and its contribution to the total matter content, at the present time, becomes negligible. For instance, in the case  $t_T = 9.50 \times 10^8 t_P$  we have  $x_T = -4.20 \times 10^{-18} m_P$  and  $\rho_{x,0} = 0.2 \times 10^{-2} \rho_{M,0}$ .

Let us now analyze in more detail the solution obtained

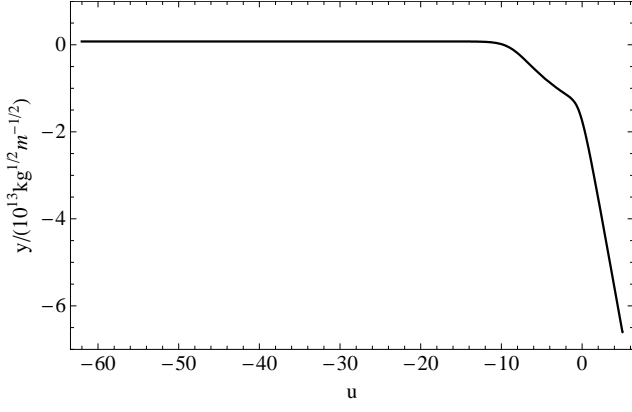


FIG. 4: Time evolution of the  $y$ -field dark energy. This field, which is practically constant during most of the second stage of evolution, became dominant only in a recent past.

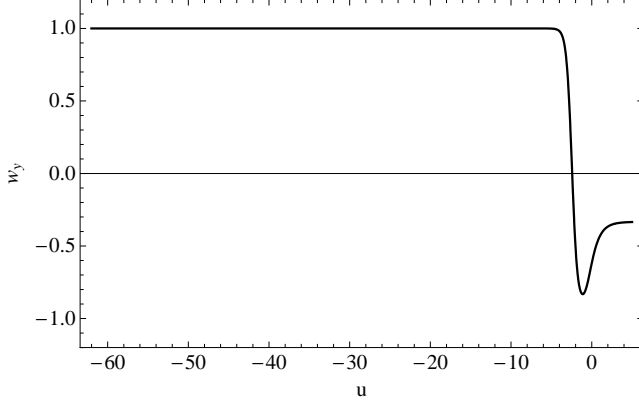


FIG. 5: Time evolution of the equation-of-state parameter for dark energy,  $w_y$ , during the second stage of evolution. At the present time,  $u_0 = 0$ , the value of this parameter is about  $-0.62$ . In the future, it will approach the value  $-1/3$ .

for  $t_T = 8.90 \times 10^8 t_P$ . In this case, at the time of transition between the first and second stages of evolution, we have  $a_T = 9.85 \times 10^{28}$ ,  $x_T = -4.45 \times 10^{-17} m_P$ ,  $y_T = 2.09 \times 10^{-2} m_P$ ,  $\rho_{x,T} = 1.16 \times 10^{-43} m_P^4 = 5.96 \times 10^{53} \text{ kg/m}^3$ , and  $\rho_{r,T} = 4.89 \times 10^{-20} m_P^4 = 2.52 \times 10^{77} \text{ kg/m}^3$ . Taking into account that  $\rho_{r,0} = 8.92 \times 10^{-128} m_P^4 = 4.6 \times 10^{-31} \text{ kg/m}^3$ , we obtain  $u_T = (1/4) \ln(\rho_{r,0}/\rho_{r,T}) = -62.02$ , and, from Eq. (29),  $C = 1.68 \times 10^{-124} m_P^4 = 7.80 \times 10^{-11} \text{ kg m}^{-1} \text{ s}^{-2}$ . The constants  $A$  and  $B$  are chosen in order to satisfy the observational constraints (35) and (36), namely,  $A = 3.16 \times 10^{-122} m_P^4 = 1.47 \times 10^{-8} \text{ kg m}^{-1} \text{ s}^{-2}$  and  $B = 4.73 \times 10^{-124} m_P^4 = 2.19 \times 10^{-10} \text{ kg m}^{-1} \text{ s}^{-2}$ . In this case, we obtain that the  $x$ -field dark matter contributes about 6% to the total matter content of the universe at the present time,  $\rho_{x,0} = 6.15 \times 10^{-2} \rho_{M,0}$ . The time evolution of the quantities  $y$ ,  $w_y$ ,  $q$ ,  $\Omega_r$ ,  $\Omega_M$ , and  $\Omega_y$ , defined in Eqs. (41)–(45), is shown in Figs. 4–7 for the case under consideration.

To finish this section, let us comment on the evolution of the scale factor today and in the future. As can

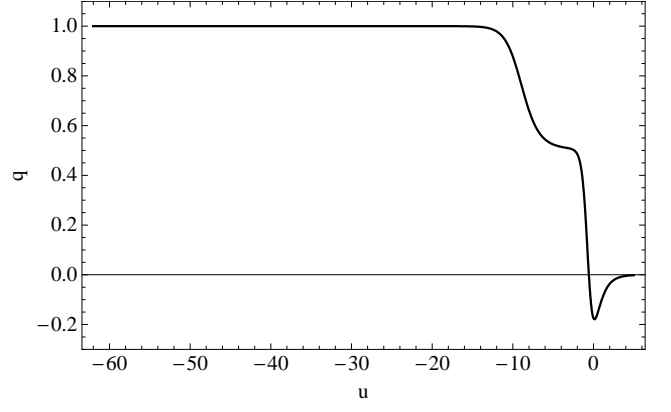


FIG. 6: Time evolution of the deceleration parameter,  $q$ , during the second stage of evolution. This parameter has its minimum (negative) value around the present time and approaches zero in the future.

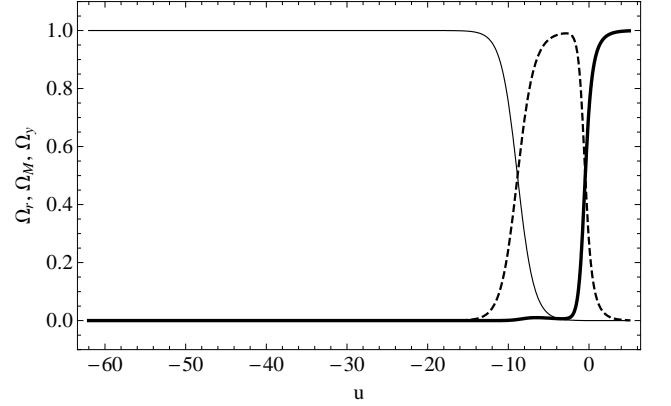


FIG. 7: Time evolution of the density parameters  $\Omega_r$  (thin curve),  $\Omega_M$  (dashed curve), and  $\Omega_y$  (thick curve) during the second stage of evolution. In this case, which corresponds to a duration of the first stage of evolution of  $8.90 \times 10^8 t_P$ , the density parameter for dark energy becomes non-negligible only recently. At the present time,  $u_0 = 0$ , the density parameters for radiation,  $\Omega_{r,0} = 4.8 \times 10^{-5}$ , matter,  $\Omega_{M,0} = 0.27$ , and dark energy,  $\Omega_{y,0} = 0.73$ , are in good agreement with observational data. The  $x$ -field dark matter contributes only about 6% to the total matter content of the universe.

be seen in Fig. 5, the equation-of-state parameter for dark matter,  $w_y$ , is today about  $-0.62$ , implying that  $a(t) \propto t^{1.75}$ , if the matter and radiation contributions are neglected. However, there is today a non-negligible contribution from matter ( $\Omega_{M,0} = 0.27$ ), which if taken alone would imply  $a(t) \propto t^{2/3}$ . Taking into account both contributions, from dark energy and from matter, we obtain that today the expansion of the universe is slightly accelerated. Naively it would seem that this acceleration should increase in the future, since the density parameter for matter will approach zero and dark energy will dominate the dynamics of evolution of the universe. However, at the same time as  $\Omega_M \rightarrow 0$ , the equation-of-state parameter for dark energy,  $w_y$ , approaches  $-1/3$  (see Fig. 5).

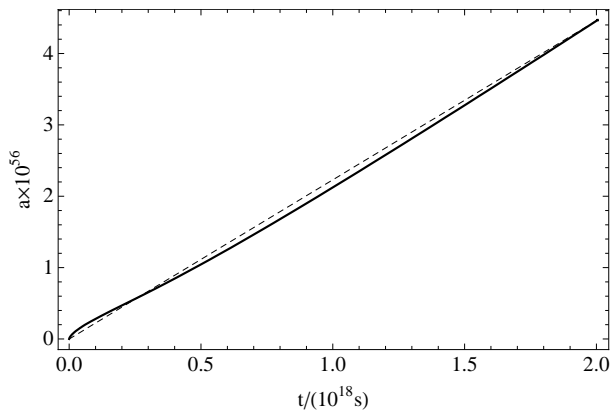


FIG. 8: Time evolution of the scale factor  $a$  (thick curve). To guide the eye, a linear growth is also represented (dashed straight line). The transition to an accelerated expansion takes place at about  $(2 - 3) \times 10^{17}$  s. At the present time,  $t \approx 4 \times 10^{17}$  s, the expansion of the universe is slightly accelerated. In the future, the scale factor growth approaches a linear behavior, a consequence of the fact that the equation-of-state parameter for dark energy approaches  $-1/3$ .

As a result, the deceleration parameter,  $q$ , approaches zero (see Fig. 6) and the scale factor grows linearly in the future. In Fig. 8 we show the evolution of the scale factor into the future, obtained by direct numerical integration of Eqs. (32) and (34), using the same initial conditions and values of the parameters  $A$ ,  $B$ , and  $C$  as in Figs. 4–7.

## V. GRAVITATIONAL WAVES

In this section, we calculate the full gravitational-wave spectrum of the Salam-Sezgin cosmological model using the formalism of the continuous Bogoliubov coefficients. This formalism was first applied to particle production in an expanding universe by Parker [17] and later extended to the case of gravitons by Henriques [18] and Moorhouse, Henriques, and Mendes [19]. It avoids, in a natural way, overproduction of gravitons of high frequencies [20]. In recent years, the formalism of continuously evolving Bogoliubov coefficients was applied to the calculation of gravitational-wave spectra in several cosmological models, revealing interesting features in the MHz/GHz range of frequencies due to the transition between the inflationary and the radiation-dominated eras [21, 22, 23].

Let us define the gravitational-wave spectral energy density parameter as

$$\Omega_{\text{GW}} \equiv \frac{1}{\rho_c} \frac{d\rho_{\text{GW}}}{d\ln\omega} = \frac{8\hbar G}{3\pi c^5 H^2} \omega^4 \beta^2,$$

where  $\rho_{\text{GW}} c^2$  and  $\omega$  are the energy density and angular frequency of the gravitational waves, respectively,  $\rho_c c^2$  is the critical energy density of the universe,  $H$  is the Hubble parameter, and  $\beta$  is a Bogoliubov coefficient, such

that  $\beta^2$  gives the number of gravitons. All quantities in the above expressions are evaluated at the present time,  $u_0 = 0$ . The squared Bogoliubov coefficient  $\beta^2$  is given by  $\beta^2 = (X - Y)^2/4$ , where  $X$  and  $Y$  are continuous functions of time, determined during the first stage of evolution by the set of differential equations

$$\dot{X} = -i\omega_0 \left(\frac{a_0}{a}\right) Y, \quad (46)$$

$$\dot{Y} = -\frac{i}{\omega_0} \frac{a}{a_0} \left[ \omega_0^2 \left(\frac{a_0}{a}\right)^2 - \frac{\ddot{a}}{a} - \left(\frac{\dot{a}}{a}\right)^2 \right] X, \quad (47)$$

where  $\ddot{a}/a$  and  $(\dot{a}/a)^2$  are given by Eqs. (18) and (19), respectively, and during the second stage of evolution by the set of differential equations

$$X_u = -i\omega_0 \left(\frac{a_0}{a}\right) \frac{Y}{\dot{a}/a}, \quad (48)$$

$$Y_u = -\frac{i}{\omega_0} \frac{a}{a_0} \left[ \omega_0^2 \left(\frac{a_0}{a}\right)^2 - \frac{\ddot{a}}{a} - \left(\frac{\dot{a}}{a}\right)^2 \right] \frac{X}{\dot{a}/a}, \quad (49)$$

where  $\ddot{a}/a$  and  $(\dot{a}/a)^2$  are given by Eqs. (39) and (40), respectively. The scale factor at the present time is given by  $a_0 = a_{\text{T}} e^{-u_{\text{T}}}$ , where the subscript T stands for the time of transition between the first and second stages of evolution.

The above differential equations are integrated numerically, from the beginning of the inflationary period till the present time. We assume that prior to the inflationary period the universe was radiation-dominated and, therefore, no gravitons were created, i.e.,  $\alpha^2(t) = 1$  and  $\beta^2(t) = 0$  for  $t \leq t_i$ , where  $\alpha$  is a Bogoliubov coefficient related to  $X$  and  $Y$  through  $\alpha^2 = (X + Y)^2/4$ . The appropriate initial conditions for the functions  $X$  and  $Y$  are then  $X(t_i) = 1$  and  $Y(t_i) = 1$ . The gravitational-wave angular frequency at present time,  $\omega_0$ , is taken to vary from about  $10^{-17}$  rad/s (corresponding to a wavelength equal, today, to the Hubble distance) to about  $10^8$  rad/s (corresponding to a wavelength equal to the Hubble distance at the end of the inflationary period).

The full gravitational-wave spectrum for the Salam-Sezgin cosmological model is shown in Fig. 9. It was calculated for the particular case of the model with  $t_{\text{T}} = 8.90 \times 10^8 t_{\text{P}}$ , discussed in detail at the end of Sect. IV.

For angular frequencies corresponding to the present size of the Hubble horizon, an upper limit on the gravitational-wave spectral energy density parameter can be derived from measurements of the cosmic microwave background radiation, namely,  $\Omega_{\text{GW}} < 1.4 \times 10^{-10}$  for  $\omega_{\text{hor}} = 1.4 \times 10^{-17}$  rad/s [24]. The gravitational-wave spectrum of the Salam-Sezgin cosmological model satisfies this constraint easily.

In the intermediate range of frequencies, from about  $10^{-15}$  rad/s to about  $10^6$  rad/s, the gravitational-wave spectrum differs substantially from the ones obtained, with the same formalism, in Refs. [21, 22, 23], in that it has a non-constant slope. This is due to the fact that, in

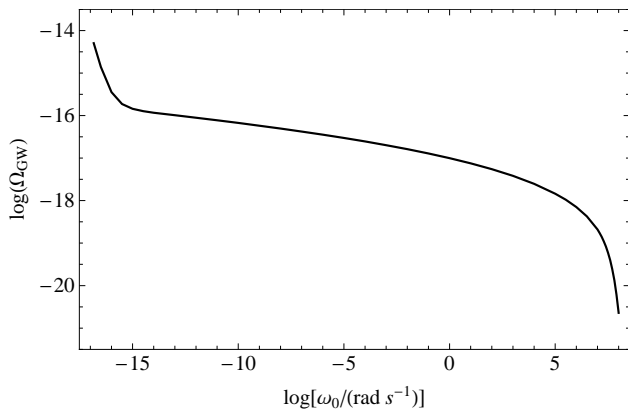


FIG. 9: Gravitational wave spectrum of the Salam-Sezgin cosmological model.

previous investigations, numerical evaluation of the Bogoliubov coefficients started at the end of inflation and, moreover, it was assumed, for the purpose of determining the initial conditions for the functions  $X$  and  $Y$ , that the inflationary period was either exponential or of the power-law type. This assumption was needed since exact analytical solutions of Eqs. (46) and (47) are known only for these types of inflation. As a result, the gravitational-wave spectrum had, in the intermediate range of frequencies, zero slope [21, 22] or constant negative slope [23], respectively. In this article, we have improved our calculation by numerically evaluating the Bogoliubov coefficients from the very beginning of the inflationary period. This approach allows us to consider any type of inflation, not just exponential or power-law. This is rather convenient for the cosmological model under consideration, since, during inflation, the equation-of-state parameter  $w = p/\rho$  changes smoothly from  $-1$  to  $-1/3$  (see Fig. 10), i.e., inflation is neither exponential nor power-law.

The shape of the spectrum in the intermediate range of frequencies can be understood as follows. It is known that, for power-law inflationary models, the spectrum has constant negative slope in this range of frequencies [25] (see also Ref. [23] for the derivation of this result using the formalism of continuous Bogoliubov coefficients). In the limit where power-law inflation approaches exponential inflation ( $s \rightarrow +\infty$ , where  $a(t) \propto t^s$ ), the slope of the spectrum approaches zero. Now, let us regard the evolution of our cosmological model, during inflation, as a succession of short periods of time, with duration  $\Delta t$ , such that in each of them inflation can be considered to be of the power-law type with  $s = \frac{2}{3}(w+1)^{-1} \approx \text{constant}$ . Gravitational waves that cross the Hubble horizon during the first of these periods, have today low frequencies. Since during this period the equation-of-state parameter is  $w = p/\rho \approx -1$  (see Fig. 10), corresponding to  $s \gg 1$ , the spectrum, for such low frequencies, is almost flat. During the second period of time,  $w$  is slightly higher and the spectrum has a small negative slope in the corresponding frequency range. Continuing with this process,

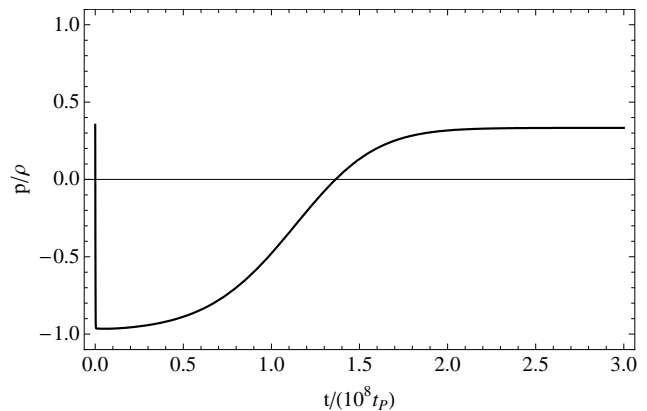


FIG. 10: Time evolution of the equation-of-state parameter,  $w = p/\rho$ , during the first stage of evolution.

we obtain a spectrum that is a succession of line segments with increasing (in modulus) slope as the frequency of the gravitational waves increases. Taking the limit  $\Delta t \rightarrow 0$ , we obtain, in the intermediate region of the spectrum, the shape shown in Fig. 9.

The gravitational-wave spectrum of the Salam-Sezgin cosmological model shows no structure in the MHz/GHz frequency range. Such a structure is expected in cosmological models in which standard (cold) inflation is followed by a preheating and/or reheating stage [21, 22, 23]. In such models, after the inflationary period the inflaton field oscillates about the minimum of its potential, leading to particle production and an increase of the energy density of radiation. The structure of the gravitational-wave spectrum in the MHz/GHz range of frequencies depends on the form of the inflationary potential near its minimum. More specifically, if the potential near its minimum is proportional to  $\phi^n$ , then  $\Omega_{\text{GW}}$  increases with the increase of  $n$  [23]. Contrarily to these standard (cold) inflationary cosmological models, we have assumed that, within the Salam-Sezgin cosmological model, inflation is of the warm type. That is, the energy density of the  $x$  and  $y$ -fields is continuously transferred to the radiation fluid and, consequently, the energy density of radiation is not diluted during inflation. Because the energy density of radiation decreases slower than the energy density of the  $x$  and  $y$ -fields (see Fig. 1), the transition from inflation to the radiation-dominated era occurs smoothly, well before the  $x$ -field starts to oscillate about the minimum of its potential. Therefore, the coherent oscillations of the  $x$ -field leave no imprint in gravitational-wave spectrum in the MHz/GHz range of frequencies.

## VI. CONCLUSIONS

In this work we have extended the analysis by Anchordoqui, Goldberg, Nawata and Nuñez based on the Salam-Sezgin supergravity theory, by including the inflationary

period in the evolution. The Salam-Sezgin potential contains two scalar fields,  $x$  and  $y$ . Assuming sufficient dissipative coupling, it follows that a period of warm inflation takes place, driven by the  $x$ -field, during which energy is being transferred continuously from both scalars to a radiation fluid. Well after the end of the inflationary period, with the universe already dominated by the energy density of the radiation fluid, the dissipative coefficients are switched off and the  $x$ -field begins a period of quick oscillations around the minimum of the potential, behaving like cold dark matter with a varying mass. The  $y$ -field, which behaves like dark energy, becomes then practically constant until very near the present epoch, when it begins to dominate the energy content of the universe. We have shown, therefore, that within the Salam-Sezgin cosmological model is possible to obtain a unified description of inflation, dark matter, and dark energy. For an appropriate choice of the parameters of the model, we were able to account for the observed values of radiation, matter, and dark-energy density parameters. However, only approximately 6% of the total matter content of the universe was obtained from the  $x$ -field, while, according to observational data, one would expect dark matter to

amount to about 80% of the total matter content. Even if not entirely successful in this respect, it is not unreasonable to expect that more sophisticated models, based on supergravity and superstring theories, could be found that may generate enough dark matter and dark energy in a fully unified description.

We have also calculated the full gravitational-wave spectrum of the Salam-Sezgin cosmological model. Using the formalism of the continuous Bogoliubov coefficients, we have shown that this spectrum has a non-constant negative slope in the frequency range ( $10^{-15} - 10^6$ ) rad/s, and that, contrarily to cosmological models in which standard (cold) inflation is followed by a preheating and/or reheating stage, the gravitational-wave spectrum shows no structure in the MHz/GHz range of frequencies.

### Acknowledgments

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